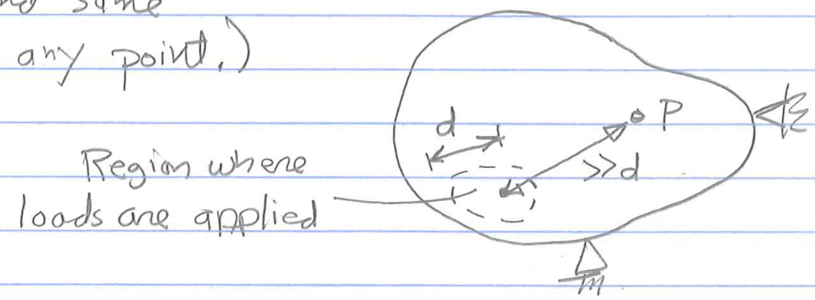
7. St. Venant's principle

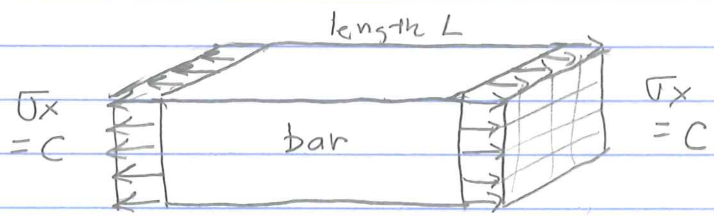
Consider a point P in a body that is far away from where loads are applied. SVP says that the stresses and strains at P will be the same for different load sets as long as the load sets are statically equivalent (same resultant force and same resultant moment about any point.)



Example:

Linearly elastic, homogeneous, isotropic bar of uniform cross-section (area A)

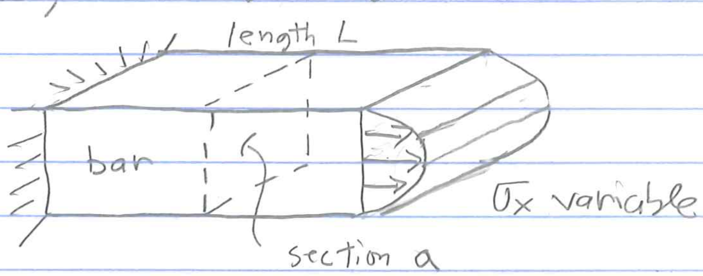
First load set consists of constant stress $\sigma_x = C$ applied over the ends



In this case, the internal stresses and strains are constant.

exact solution
$$\begin{cases} \sigma_x = C, \quad \sigma_y = \sigma_z = \tau_{xy} = \tau_{yz} = \tau_{xz} = 0 \\ \varepsilon_x = C/E, \quad \varepsilon_y = \varepsilon_z = -\nu C/E, \quad \gamma_{xy} = \gamma_{yz} = \gamma_{xy} = 0 \end{cases}$$

Second load set consists of spatially varying stresses at the right end and support stresses at the left end.



If the two loads sets at the right end are statically equivalent (both equivalent to a concentric force $P = CA$ acting to the right), then the two load sets at the left end will also be statically equivalent. Why?

Thus, by SVP, if section a is far enough away from the ends, then the stresses there due to the second set of loads can be approximated by the stresses from the first load set.

If the bar is long compared to its cross-section dimensions, we could assume that the uniform stress state predominates for the second load set. With this assumption, the deflection of the right end can be approximated by

$$\delta_{\text{right end}} = \frac{P}{A} \frac{1}{E} \cdot L = P/\frac{AE}{L}.$$

Written as $\dfrac{P}{\delta_{\text{right end}}} = \dfrac{AE}{L}$, the term $\dfrac{AE}{L}$ is recognized as the axial stiffness of the bar.

The constant stress and strain solution for the first load set can be referred to as the St. Venant solution for axial loading of a uniform bar (linearly elastic, homogenous, isotropic material)